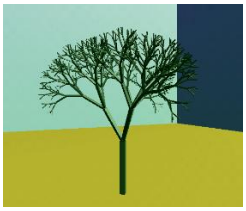

GATS Companion to Calendars

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Overview

A companion to calendar conversion, inspired by Jean Meeus' book Astronomical Algorithms.

Which calendars and systems do we cover?

- Gregorian (*Civil*) Calendar
- Hebrew Calendar
- Julian Date
- Julian Calendar
- Islamic Calendar
- Vulcan Calendar

The Math…

All calculations are real number operations unless specifically stated otherwise.

div indicates integer division

mod indicates integer modulus

Utility formulae

Floating-point modulus

$$\text{mod}(a,b) \Longrightarrow a - b\left\lfloor\frac{a}{b}\right\rfloor$$

Integer modulus

a* mod *b is the standard modulus in computing (e.g., a % b in C/C++/Java/etc.)

Time-of-day formulae

$tod \in \mathbb{R}, where\ 0 \le tod < 1$

Time-of-day from hours, minutes, seconds

$$\text{tod}(second, minute, hour) \Longrightarrow \frac{second + 60 \cdot (minute + 60 \cdot hour)}{24 \cdot 60 \cdot 60}$$

Hours, minutes, seconds, from time-of-day

$$\text{hms}(tod) \Longrightarrow \langle hours \quad minutes \quad seconds \rangle$$

$secondsInDay \leftarrow \lfloor tod \cdot 24 \cdot 60 \cdot 60 + 0.5 \rfloor$
$hours \leftarrow secondsInDay$ **div** $(60 \cdot 60)$
$minutes \leftarrow (secondsInDay$ **div** $60)$ **mod** 60

$seconds \leftarrow secondsInDay$ **mod** 60

Julian Date (JD) and Julian Day Number (JDN) formulae

Any operation that takes a JD can take a JDN without modification.

Day of week from Julian Date (JD)

$\text{DayOfWeek}(jd) \Longrightarrow \text{mod}(jd + 0.5, 7)$

Time-of-day from Julian Date (JD)

$\text{tod}(jd) \Longrightarrow (jd + 0.5) - \lfloor jd + 0.5 \rfloor$

HMS from Julian Date (JD)

$\text{hms}(jd) \Longrightarrow \text{hms}(tod)$

$tod \leftarrow (jd + 0.5) - \lfloor jd + 0.5 \rfloor$

Julian Day Number (JDN) from Julian Date (JD)

The JDN is the JD adjusted to 12 pm (12:00:00) of that given day.

$\text{jdn}(jd) \Longrightarrow \lfloor jd + 0.5 \rfloor$

In some interpretations, the JDN is the JD adjusted to 12 am (0:00:00) of that given day.

$\text{jdn}(jd) \Longrightarrow \lfloor jd + 0.5 \rfloor - 0.5$

Gregorian calendar formulae

$year\ is\ an\ astronomical\ year\ number$
$month \in [1..12]$
$day \in [1..31]$
$wday \in [0..6], where\ 0 = Monday$
$GregorianEpoch = 1{,}721{,}425.5$

Gregorian (G) Leap-year

$\text{leapyear}(year) \Longrightarrow (year\ \textbf{mod}\ 4 = 0)\ \textbf{and}\ (year\ \textbf{mod}\ 100\ \neq 0)\ \textbf{or}\ (year\ \textbf{mod}\ 400 = 0)$

Gregorian (G) add months (n)

$\text{gregorianAddMonths}(year, month, day, n) \Longrightarrow \langle y \quad m \quad d \rangle$

$y \leftarrow year + n\ \textbf{div}\ 12$
$m \leftarrow month + n\ \textbf{mod}\ 12$
$adjust \leftarrow (m - 1)\ \textbf{div}\ 12 + (m - 12)\ \textbf{div}\ 12$
$y \leftarrow y + adjust$
$m \leftarrow m - adjust \cdot 12$
$d \leftarrow \min\left(day, daysInMonth\left(m, leapYear(y)\right)\right)$

Gregorian (G) add years (n)

$\text{gregorianAddYears}(year, month, day, n) \Longrightarrow \langle y \quad m \quad d \rangle$

$y \leftarrow year + n$

$m \leftarrow month$
$d \leftarrow day$
$(m = 2 \textbf{ and } d = 29 \textbf{ and } leapyear(y) \neq true) \Longrightarrow (d = 28)$

Gregorian to JDN (Integer implementation)

jdn$(year, month, day) \Longrightarrow jdn$

$a \longleftarrow (14 - month) \textbf{ div } 12$
$y \longleftarrow year + 4800 - a$
$m \longleftarrow month + 12a - 3$
$jdn \longleftarrow day + (153 \cdot m + 2) \textbf{ div } 5 + 365 \cdot y + y \textbf{ div } 4 - y \textbf{ div } 100 + y \textbf{ div } 400 - 32045.5$

Gregorian to JDN (Floating-point implementation)

All variables are real numbers

$y \longleftarrow year - 1$

$$leapAdjust \longleftarrow \begin{cases} 0, & where\ month \leq 2 \\ -1, & where\ month > 2\ and\ year\ is\ a\ leapyear \\ -2, & where\ month > 2\ and\ year\ is\ not\ a\ leapyear \end{cases}$$

$$jdn \longleftarrow GregorianEpoch - 1 + 365y + \left\lfloor \frac{y}{4} \right\rfloor - \left\lfloor \frac{y}{100} \right\rfloor + \left\lfloor \frac{y}{400} \right\rfloor + \left\lfloor \frac{367month - 362}{12} + leapAdjust + day \right\rfloor$$

JD to Gregorian (Integer implementation)

gregorian$(jd) \Longrightarrow \langle year \quad month \quad day \rangle$

$jd \leftarrow \lfloor jd - 0.5 \rfloor + 0.5$
$a \leftarrow \lfloor jd + 0.5 \rfloor + 32044$
$b \leftarrow (4 \cdot a + 3) \textbf{ div } 146097$
$c \leftarrow a - (146097 \cdot b) \textbf{div } 4$
$d \leftarrow (4 \cdot c + 3) \textbf{ div } 1461$
$e \leftarrow c - 1461 \cdot d \textbf{ div } 4$
$m \leftarrow (5 \cdot e + 2) \textbf{ div } 153$
$day \leftarrow e - (153 \cdot m + 2) \textbf{ div } 5 + 1$
$month \leftarrow m + 3 - 12 \cdot (m \textbf{ div } 10)$
$year \leftarrow 100 \cdot b + d - 4800 + m \textbf{ div } 10$

Gregorian to JD (with H:M:S)

jd$(year, month, day, hour, minute, second) \Longrightarrow jd$

$jdn \leftarrow$ jdn$(year, month, day)$
$tod \leftarrow$ tod$(hour, minute, second)$
$jd \leftarrow jdn + tod$

JD to Gregorian (with H:M:S)

gregorianWithHMS$(jd) \Longrightarrow \langle year \quad month \quad day \quad hour \quad minute \quad second \rangle$

$\langle year \quad month \quad day \rangle \leftarrow$ gregorian(jd)
$\langle hour \quad minute \quad second \rangle \leftarrow$ hms(jd)

Julian calendar formulae

$year$ *is an astronomical year number*
$month \in \{1, 2, 3, \cdots, 12\}$
$day \in \{1, 2, 3, \cdots, 31\}$
$wday \in \{0, 1, 2, 3, \cdots, 6\}, where\ 0 = Monday$
$JulianEpoch = 1{,}721{,}423.5$

Julian (J) Leap-year

leapyear$(year) \Longrightarrow (year \ \mathbf{mod}\ 4 = 0)$

Julian (J) add months (n)

julianAddMonths$(year, month, day, n) \Longrightarrow \langle y \quad m \quad d \rangle$

$y \leftarrow year + \ n \ \mathbf{div}\ 12$
$m \leftarrow month + n \ \mathbf{mod}\ 12$
$adjust \leftarrow (m - 1) \ \mathbf{div}\ 12 + (m - 12) \ \mathbf{div}\ 12$
$y \leftarrow y + adjust$
$m \leftarrow m - adjust \cdot 12$
$d \leftarrow \min\left(day, daysInMonth\left(m, leapYear(y)\right)\right)$

Julian (J) add years (n)

julianAddYears$(year, month, day, n) \Longrightarrow \langle y \quad m \quad d \rangle$

$y \leftarrow year + \ n$
$m \leftarrow month$
$d \leftarrow day$
$(m = 2 \ \mathbf{and}\ d = 29 \ \mathbf{and}\ leapyear(y) \neq true) \Longrightarrow (d \leftarrow 28)$

Julian to JDN (Integer implementation)

jdn$(year, month, day) \Longrightarrow jdn$

$a \longleftarrow (14 - month) \ \mathbf{div}\ 12$
$y \longleftarrow year + 4800 - a$
$m \longleftarrow month + 12a - 3$
$jdn \longleftarrow day + (153 \cdot m + 2) \ \mathbf{div}\ 5 + 365 \cdot y + y \ \mathbf{div}\ 4 - 32083.5$

JD to Julian

julian$(jd) \Longrightarrow \langle year \quad month \quad day \rangle$

$a \leftarrow \lfloor jd + 0.5 \rfloor$
$b \leftarrow a + 1524$
$c \leftarrow \left\lfloor \frac{b - 122.1}{365.25} \right\rfloor$
$d \leftarrow \lfloor 365.25 \cdot c \rfloor$
$e \leftarrow \left\lfloor \frac{b - d}{30.6001} \right\rfloor$
$month \leftarrow \left\lfloor \begin{cases} e < 14 \Longrightarrow e - 1 \\ e \not< 14 \Longrightarrow e - 13 \end{cases} \right\rfloor$
$year \leftarrow \left\lfloor \begin{cases} month > 2 \Longrightarrow c - 4716 \\ month \not> 2 \Longrightarrow c - 4715 \end{cases} \right\rfloor$
$day \leftarrow b - d - \lfloor 30.6001 \cdot e \rfloor$

Julian to JD (with H:M:S)

$\text{jd}(year, month, day, hour, minute, second) \Longrightarrow jd$

$jdn \leftarrow \text{jdn}(year, month, day)$
$tod \leftarrow \text{tod}(hour, minute, second)$
$jd \leftarrow jdn + tod$

JD to Julian (with H:M:S)

$\text{julianWithHMS}(jd) \Longrightarrow \langle year \quad month \quad day \quad hour \quad minute \quad second \rangle$

$\langle year \quad month \quad day \rangle \leftarrow \text{julian}(jd)$
$\langle hour \quad minute \quad second \rangle \leftarrow \text{hms}(jd)$

Islamic calendar formulae

$year$ *is an astronomical year number*
$month \in [1..12]$
$day \in [1..30]$
$wday \in [0..6], where\ 0 = al\text{–}'ahad, 1 = al\text{–}'ithnayn, 2 = alth - thalatha,$
$3 = al\text{–}'arb`a', 4 = al - khamis, 5 = al - jum`a, 6 = as - sabt$
$IslamicEpoch = 1{,}948{,}439.5$

Calendar dates run from noon to noon.

Weekday names are used from midnight to midnight.

Islamic (I) Leap-year

$\text{leapyear}(year) \Longrightarrow (11 \cdot year + 14)\ mod\ 30 < 11$

Islamic (I) Days in month

$$\text{dayInMonth}(month, isLeapyear) \Longrightarrow \begin{cases} 30, if\ (month\ is\ odd)\ \textbf{or}\ (month = 12\ \textbf{and}\ isLeapyear) \\ 29\ otherwise \end{cases}$$

Islamic (J) add months (n)

$\text{islamicAddMonths}(year, month, day, n) \Longrightarrow \langle y \quad m \quad d \rangle$

$y \leftarrow year + n\ \textbf{div}\ 12$
$m \leftarrow month + n\ \textbf{mod}\ 12$
$adjust \leftarrow (m - 1)\ \textbf{div}\ 12 + (m - 12)\ \textbf{div}\ 12$
$y \leftarrow y + adjust$
$m \leftarrow m - adjust \cdot 12$
$d \leftarrow \min\left(day, daysInMonth\left(m, leapYear(y)\right)\right)$

Islamic (I) add years (n)

$\text{islamicAddYears}(year, month, day, n) \Longrightarrow \langle y \quad m \quad d \rangle$

$y \leftarrow year + n$
$m \leftarrow month$
$d \leftarrow day$
$(m = 12\ \textbf{and}\ d = 30\ \textbf{and}\ leapyear(y) \neq true) \Longrightarrow (d \leftarrow 29)$

Islamic (I) to JDN

$\text{jdn}(year, month, day) \Longrightarrow jdn$

$$jdn \longleftarrow day + \lceil 29.5 \cdot (month - 1) \rceil + (year - 1) \cdot 354 + \left\lfloor \frac{3 + 11 \cdot year}{30} \right\rfloor + IslamicEpoch - 1$$

JD to Islamic

$\text{islamic}(jd) \Longrightarrow \langle year \quad month \quad day \rangle$

$$jd \leftarrow \lfloor jd \rfloor + 0.5$$
$$year \leftarrow \left\lfloor \frac{30 \cdot (jd - IslamicEpoch) + 10646}{10631} \right\rfloor$$
$$month \leftarrow \min\left(12, \left\lceil \frac{jd - (29 + \text{jdn}(year, 1, 1))}{29.5} \right\rceil + 1\right)$$
$$day \leftarrow jd - \text{jdn}(year, month, 1) + 1$$

Islamic to JD (with H:M:S)

$\text{jd}(year, month, day, hour, minute, second) \Longrightarrow jd$

$$jdn \leftarrow \text{jdn}(year, month, day)$$
$$tod \leftarrow \text{tod}(hour, minute, second)$$
$$tod \geq 0.5 \Longrightarrow tod \leftarrow tod - 1$$
$$jd \leftarrow jdn + tod$$

JD to Islamic (with H:M:S)

$\text{islamicWithHMS}(jd) \Longrightarrow \langle year \quad month \quad day \quad hour \quad minute \quad second \rangle$

$$\langle year \quad month \quad day \rangle \leftarrow \text{islamic}(jd)$$
$$\langle hour \quad minute \quad second \rangle \leftarrow \text{hms}(jd)$$

Hebrew calendar formulae

$year$ *is an astronomical year number*
$month \in \{1, 2, 3, \cdots, 13\}$
$day \in \{1, 2, 3, \cdots, 30\}$
$HebrewEpoch = 347{,}995.5$

Calendar dates run from noon to noon.

Weekday names are used from midnight to midnight.

Hebrew (H) leap-year

$\text{leapyear}(year) \Longrightarrow \text{mod}(7 \cdot year + 1, 19) < 7$

Hebrew (H) months in year

$\text{monthsInYear}(year) \Longrightarrow months$

$$months \leftarrow 12$$
$$leapyear(year) \Longrightarrow months \leftarrow months + 1$$

Hebrew (H) delay of week

$\text{delayOfWeek}(year) \Longrightarrow days$

$$months \leftarrow \left\lfloor \frac{235 \cdot year - 234}{19} \right\rfloor$$
$$parts \leftarrow 12084 + 13753 \cdot months$$
$$days \leftarrow months \cdot 29 + \left\lfloor \frac{parts}{25920} \right\rfloor$$
$$\text{mod}(3 \cdot (days + 1), 7) < 3 \Rightarrow days \leftarrow days + 1$$

Hebrew (H) delay adjacent year

delayAdjacentYear$(year) \Rightarrow days$

$$last \leftarrow \text{delayOfWeek}(year - 1)$$
$$present \leftarrow \text{delayOfWeek}(year)$$
$$next \leftarrow \text{delayOfWeek}(year + 1)$$
$$days \leftarrow 0$$
$$next - present = 356 \Rightarrow days \leftarrow 2$$
$$(next - present \neq 356) \textbf{ and } (present - last = 382) \Rightarrow days \leftarrow 1$$

Hebrew (H) days in year

daysInYear$(year) \Rightarrow \text{jdn}(year + 1,7,1) - \text{jdn}(year, 7,1)$

Hebrew (H) days in month

daysInMonth$(year, month) \Rightarrow days$
$month = 2$ **or** $month = 4$ **or** $month = 6$ **or** $month = 10$ **or** $month = 13 \Rightarrow days \leftarrow 29$
$month = 12$ **and** $leapyear(year) \neq true \Rightarrow days \leftarrow 29$
$month = 8$ **and** $\text{mod}(\text{daysInYear}(year), 10) \neq 5 \Rightarrow days \leftarrow 29$
$month = 9$ **and** $\text{mod}(\text{daysInYear}(year), 10) = 3 \Rightarrow days \leftarrow 29$
$days \leftarrow 30$

Hebrew (H) to JDN

jdn$(year, month, day) \Rightarrow jdn$

$$jdn \leftarrow HebrewEpoch + \text{delayOfWeek}(year) + \text{delayAdjacentYear}(year) + day + 1$$
$$month < 7 \Rightarrow \begin{matrix} for\ each\ m \in [7..monthInYear(year)], jdn \leftarrow jdn + \text{daysInMonth}(year, m) \\ for\ each\ m \in [1..month - 1], jdn \leftarrow jdn + \text{daysInMonth}(year, m) \end{matrix}$$
$$month \geq 7 \Rightarrow for\ each\ m \in [7..month - 1], jdn \leftarrow jdn + \text{daysInMonth}(year, m)$$

JD to Hebrew

hebrew$(jd) \Rightarrow \langle year \quad month \quad day \rangle$

$$jd \leftarrow \lfloor jd \rfloor + 0.5$$
$$count \leftarrow \left\lfloor \frac{(jd - HebrewEpoch) \cdot 98496}{35975351} \right\rfloor$$
$$year \leftarrow count - 1$$
$$i \leftarrow count$$
$$while\ jd \geq \text{jdn}(i, 7,1) \Rightarrow year \leftarrow year + 1, i \leftarrow i + 1$$
$$first \leftarrow 1$$
$$jd < \text{jdn}(year, 1,1) \Rightarrow first \leftarrow 7$$
$$month \leftarrow \text{first}$$
$$i \leftarrow first$$
$$while\ jd > \text{jdn}(year, i, \text{daysInMonth}(year, i)) \Rightarrow month \leftarrow month + 1, i \leftarrow i + 1$$

$$day \leftarrow \lfloor jd - \text{jdn}(year, month, 1) + 1 \rfloor$$

Hebrew to JD (with H:M:S)

$\text{jd}(year, month, day, hour, minute, second) \Longrightarrow jd$

$$jdn \leftarrow \text{jdn}(year, month, day)$$
$$tod \leftarrow \text{tod}(hour, minute, second)$$
$$tod \geq 0.5 \Longrightarrow tod \leftarrow tod - 1$$
$$jd \leftarrow jdn + tod$$

JD to Hebrew (with H:M:S)

$\text{islamicWithHMS}(jd) \Longrightarrow \langle year \quad month \quad day \quad hour \quad minute \quad second \rangle$

$$\langle year \quad month \quad day \rangle \leftarrow \text{hebrew}(jd)$$
$$\langle hour \quad minute \quad second \rangle \leftarrow \text{hms}(jd)$$

Star Trek Vulcan Calendar Formulae

Why Vulcan and not *Star Dates*? Because there are at least three different *Star Date* systems in the various series and the rules don't seem to be applied consistently. The Vulcan calendar however is good.

Terms

R'tas: Vulcan year (R)

T'Kuhati: Vulcan month (TK)

T'Ved: Vulcan day (TV)

V'hral: Vulcan hour (Vh)

lirt'k: Vulcan minute (lk)

lik'rt: Vulcan second (lt)

Conversions

1 R = 12 TK = 252 TV = 266.4 Earth day

1 TK = 21 TV

1 TV = 18Vh

1 Vh = 54 lk

1 lk = 54 lt

Month Names

1	Z'at
2	D'ruh
3	K'riBrax
4	re'T'Khutai
5	T'keKhuti
6	Khuti
7	Ta'Krat
8	K'ri'lior
9	et'khior
10	T'lakht

11 T’ke’Tas
12 Tasmeen

Conversions
Z’at 1, 9051 0:0:0 = December 11, 2364 12:0:0 = JD 2584838

Epoch
Z’at 1, 1 = JD 173651.6

Appendix
References
Meeus, Jean (1998) Astonomical Algorithms (2nd ed.) Willmann-Bell, Inc.

Document History
2013-10-18	Document created
2019-10-10	Vulcan calendar added
2020-10-04	Published as a *GATS Companion*
2021-10-18	Clarified Islamic days in month value
2021-10-19	Clarified Islamic new-year modulus operation
2023-10-21	Fixed missing bracket in Hebrew formula.